Research has indicated ... that 90% of the most commonly encountered texts in households require knowledge of fractions, decimals, percent, ratio and proportion.

Partitioning

Why is partitioning an important notion?

The teaching and learning of fractions, particularly in the middle years and beyond, is a key aspect of numeracy as it underpins the proportional reasoning used in much of our everyday lives. Research has indicated, for example, that 90% of the most commonly encountered texts in households require knowledge of fractions, decimals, percent, ratio and proportion.

Previously it had been assumed that once learners can identify fractions from a given diagram, can identify a simple part of a given whole, or can shade a diagram to show a given fraction, then they are ready to start renaming fractions.

However, with pre-partitioned fraction diagrams, learners are often focusing only on the number to shade in rather than attending to all relevant aspects of partitioning: the unit/whole, the number of parts in the whole, the size of each part, and the number of parts required. In fact learners need to explore examples of correct and incorrect fraction representations if they are to recognise that equal parts/equal shares are necessary. Without focusing on the connection between fractions and partitioning, learners may adopt a narrow rule-based approach resulting in difficulties in:

- reading, renaming, ordering, interpreting and applying decimal fractions and common fractions, particularly those greater than 1
- recognising the applicability of ratio and proportion and justifying this mathematically in terms of fractions, percentage or written ratios (Siemon, Virgona & Cornielle, 2001).

This paper provides guidelines on partitioning and how leaders can support educators in fostering an understanding of partitioning, both in themselves and in their learners.

What is partitioning?

Partitioning is the process of dividing an object or objects into more parts. The whole should include all of these parts, with no parts left over. When working with fractions these parts must be equal. Fractions are a difficult concept for learners to grasp as there are so many different uses for fractions and so many different contexts that they appear in.

Fractions differ from whole number in that they are relative to the whole. For example, not all halves are the same as they depend on what the whole is. In addition, other key ideas need to be established, namely the need for equal parts, the number of parts helps to name the part (eg 5 equal parts are called fifths), and that as the number of parts increases (halves to quarters) the size of the part decreases (they get smaller).

An understanding of fractions through partitioning helps to develop these ideas. Paper folding experiences in order to generate halving, thirding and fifthing strategies enable learners to create their own fraction representations.

In contrast, there is considerable evidence to suggest that learners who do not create their own fraction diagrams through these generalised partitioning strategies have difficulty making sense of template fraction diagrams and formatted number lines (Empson, Junk, Dominguez & Turner, 2006: Lamon, 2007: Wong & Evans, 2008). Counting and colouring/shading someone else's pre-partitioned diagram is merely practice for existing knowledge and also encourages learners to apply whole number-based strategies. An example of whole number thinking is where a student might identify that 1 third is larger than 1 half because three is larger than two.

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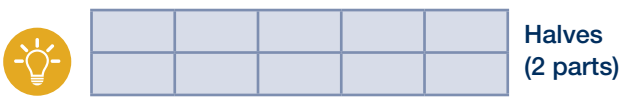
How can educators help learners understand partitioning?

In the early years, children explore quantity before learning to count. Problems in context can generate discussion and encourage learners to make meaningful comparisons. For example 'I have 4 apples and there are 5 children. How can we share these so that everyone has the same?' This provides an opportunity for learners to apply reasoning and talk about the consequences of their actions. Learners soon attach amounts to quantities to help describe them. At this stage they may not even be aware of the key ideas that are involved in more general understanding of fractions: that equal parts are involved, that parts are named according to how many there are, and that the parts decrease in size as the number of parts increase.

In terms of the halving partitioning strategy, learners do not need to be taught halving. It is an intuitive process. They should, however, be encouraged to explore halving a wide variety of wholes so that they begin to understand that fractions are relative, and may compare and contrast, noting similarities and differences. For example, square paper can be folded to form two rectangles or two triangles: two different shapes that have the same area. Different ways of making the same fractional parts support learners to attend to the aspect of area rather than shape. Successive halving experiences enable students to make and name fractions in the halving family, eg quarters, eighths, sixteenths. The thirding partitioning strategy can then be explored, in particular, noticing that a third is smaller than a half so thinking 'find half and a little bit less to make my first third, I need two more parts, so I can halve what remains to make my three equal parts'. Region models that combine partitioning strategies can show, for example, fifths (5 parts) by halves (2 parts) results in tenths (10 parts). This exploration leads to the development of generalisations such as 'As the number of parts increases, the parts get smaller' as well as fraction renaming by recognising 'for each fifth I now have two smaller parts, ten equal parts, tenths'.



(Region model)

Number lines can also be used to depict fractions by building on from the halving, thirding and fifthing strategies. These can help learners to think about a linear representation (fraction as a number that can be located on a number line) and help develop flexibility when thinking about fractions.

Following on from intuitive halving, this approach of multiple representations of fractions and the exploration of the strategies of halving, thirding and fifthing (Siemon, 2003) supports learners to develop understanding about the relative size of fractions and how they are made, named and renamed.

(Number line model)

Fractions are a difficult concept for learners to grasp, not least because they are often situated within word problems. A negative experience can impact a learner's disposition as well as their ability to understand higher mathematical and scientific concepts. This emphasises the need for learners to explore a broad range of contexts to consolidate their understanding. Part-whole models, region models, number lines and discrete fraction models (collections of things) can be used to present a wide range of contextual situations.

How can leaders support their staff?

Educators need opportunities to design learning tasks that engage learners in productive exploration. This exploration should involve learners making choices, describing, recording, explaining and justifying. Rich tasks should preferably involve elements of several of the mathematics proficiencies. Plan time for learners to discuss and collaborate in small groups to partition wholes in a purposeful way. This way of learning fosters reasoning skills and allows trial and error activity to develop deeper understanding. When learners have choices of strategy, choices of process, or both, they are more likely to create knowledge for themselves.

When educators are given opportunities to experience collaborative learning and exploration for themselves they are more likely to successfully organise classroom learning experiences in the same way. Learners also benefit from educators collaboratively developing forms of evidence and assessment that focus on the big idea of partitioning rather than moving on too quickly to formalised and superficial fraction content and assessment using pre-partitioned worksheets.

When observing lessons, look for learners who:

- can make fractional parts in different ways
- discuss and collaborate in small groups to purposefully partition wholes
- can make choices which they can describe, record, explain and justify.

Leaders should consider how they work with their staff to incorporate the big ideas in number into common agreements around planning, teaching and assessment at their site.

Reflective questions for leaders to ask their teachers

When looking at and discussing the numeracy and mathematics program, you could, for example, ask the teacher:

- How many students in your classroom understand partitioning as a way of making comparisons or solving a problem involving fractions?
- For those who seem to be struggling to understand partitioning, what steps are you taking to address this learning need?
- Have you used the partitioning common misunderstanding tools? (Siemon, 2009)
- How do you use evidence of learner thinking about partitioning to inform your planning and assessment of learner achievement?
- What opportunities are there for learners to investigate fractions, as well as communicate and share solutions and representations using digital technology?

Further resources

The big ideas in number are discussed in further detail in the following mathematics papers:

- 3.0 Conceptual understanding: Number and algebra
- 3.1 Trusting the count
- 3.2 Place value
- 3.3 Multiplicative thinking
- 3.5 Proportional reasoning
- 3.6 Generalising.

All papers in this series are based on the work of Dianne Siemon, Professor of Mathematics Education at RMIT and a key text (Siemon et al, 2015).

http://bit.ly/BestAdviceSeries

Further reading

ACER PAT Teaching Resources Centre houses relevant examples of partitioning, for example:

- multiplying a two-digit number by a one-digit number
- fraction of a group
- fraction representation – unit fractions
- equivalent fractions
- fraction of a fraction.

Schwarz VJ (2017) *Fractions: Building a strong foundation based on conceptual understanding*, Yale National Initiative, retrieved from https://teachers.yale.edu/curriculum/viewer/initiative_11.06.06_u

Siemon D (2004) 'Partitioning: The missing link in building fraction knowledge and confidence', paper presented at the *Australian Association of Mathematics Teachers virtual conference*, https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/partitioning.pdf

Van De Walle JA, Karp K & Bay-Williams JM (2016) *Elementary and Middle School Mathematics: Teaching Developmentally*, Ninth global edition, UK: Pearson Education Limited. In particular, refer to chapter on 'Algebraic thinking: Generalisations, patterns and functions'.

Victorian Department of Education and Training, Mathematics Developmental Continuum F–10 This resource provides evidence-based indicators of progress, linked to powerful teaching strategies.

Victorian Department of Education and Training, Assessment for Common Misunderstandings These tools draw on highly focussed, research-based Probe Tasks and the Probe Task Manual (RMIT), as well as a number of additional tasks and resources which have been organised to address 'common misunderstandings'.

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Siemon D, Beswick K, Brady K, Clark J, Faragher R & Warren E (2015) *Teaching Mathematics: Foundations to Middle Years*, 2nd edition, Melbourne, Oxford University Press

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This paper is part of the DECD Leading Learning Improvement *Best advice* series, which aims to provide leaders with the research and resource tools to lead learning improvement across learning areas within their site.

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